

The Logic Lab

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IM PAN



a challenge for CNK

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- ▶ These are phenomena we can *touch, see, and build*.

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- ▶ *But how do we exhibit the invisible architecture of reality?*

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- ▶ Science centers such as CNK excel at demonstrating physics, biology, and engineering.
- ▶ These are phenomena we can *touch, see, and build*.
- ▶ *But how do we exhibit the invisible architecture of reality?*
E.g., how do we put a mathematical paradox in a display case?

a challenge for CNK

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- ▶ *"This sentence is false."*

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But we can also ask how to display mathematical infinity?

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Well, displaying a paradox is not necessarily hard:

- ▶ *"This sentence is false."*

But we can also ask how to display mathematical infinity?

- ▶ *In this talk I shall discuss how to deploy a physical realization of certain abstract ideas, including the concept of infinity.*

~1870–1930

The great paradigm shift:

~1870–1930

The great paradigm shift:

- ▶ rise of modernity: *traditional structures break down.*
- ▶ shattering of absolutes: *"death of God" and great decentralization.*
- ▶ subjectivity over objectivity: *human perspective becomes central.*
- ▶ multiple foundations: *single, universal truth is replaced by many valid frameworks.*
- ▶ perspectivism over realism: *our perspective changes what we see.*
- ▶ rise of new philosophies: *relativism, structuralism, and formalism.*

~1870–1930

- ▶ physics: *theory of relativity and quantum mechanics.*
- ▶ mathematics: *discovery of logical paradoxes, and incompleteness theorem.*
- ▶ philosophy: *creation of formal artificial languages.*
- ▶ art: *fragmented reality of cubism, Bauhaus, etc.*
- ▶ psychology: *discovery of the unconscious mind.*
- ▶ ethics: *fall of universal morality.*
- ▶ nature: *chaos and probability challenge perfect regularity.*

from ancient paradoxes to mathematical crisis

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- ▶ In late 19th century, Gottlob Frege tries to "logicize" mathematics.
- ▶ In 1902, Bertrand Russell showed that the Liar paradox was not just a word game,

from ancient paradoxes to mathematical crisis

- ▶ Self-referential paradoxes date back to ancient greece.
- ▶ In late 19th century, Gottlob Frege tries to "logicize" mathematics.
- ▶ In 1902, Bertrand Russell showed that the Liar paradox was not just a word game, it was a fatal contradiction at the foundation of mathematics.



Lwów–Warsaw school

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The Lwów–Warsaw school emerged as a global powerhouse in logic, where Alfred Tarski and Jan Łukasiewicz established the rigorous mathematical definitions of truth, semantics, and many-valued logics, among other things.

Hilbert and Gödel

- ▶ “Hilbert’s program”:

Hilbert and Gödel

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Hilbert and Gödel

- ▶ “Hilbert’s program”: formalizing mathematics into strict axiomatic systems to prove theorems using "algorithms."
- ▶ In 1931, Kurt Gödel provided a definitive answer to Hilbert’s program.

Gödel's incompleteness theorems

Theorem

Any consistent (no contradiction) formal system which can formalize arithmetic is incomplete.

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Example

Mathematics!

- Within any such system, there exist true mathematical statements that cannot be proven using the system's own rules.

Gödel's incompleteness theorems

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Any consistent (no contradiction) formal system which can formalize arithmetic is incomplete.

Example

Mathematics!

- ▶ Within any such system, there exist true mathematical statements that cannot be proven using the system's own rules.
- ▶ This reframed the philosophy of mathematics, pointing out absolute mathematical limits and giving birth to modern computability theory.

a moral consequence

- ▶ The “driving agent” of mathematics will always have blind spots.

a moral consequence

- ▶ The “driving agent” of mathematics will always have blind spots.
- ▶ This is what we need to teach kids!

mathematical logic

What is mathematical logic as a field of study?

introduction
○○

a long flash back
oooooooooooo●

infinity
oooooooooo

computation
ooo

logical puzzles
○○

fairness and cooperation
oooooo

conclusion and gallery
ooooo

Proposed subjects to CNK

infinity

Core concepts: finite, countability, uncountability, continuum, bijections, Cantor's diagonalization, etc.

- ▶ *Common understanding:* infinity = unreachable, or limitless, or very very every very large/far/big

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- ▶ *Common understanding:* infinity = unreachable, or limitless, or very very every very large/far/big
- ▶ *Mathematical reality:* It is a deep subject connected to every inch of mathematical geography.

infinity

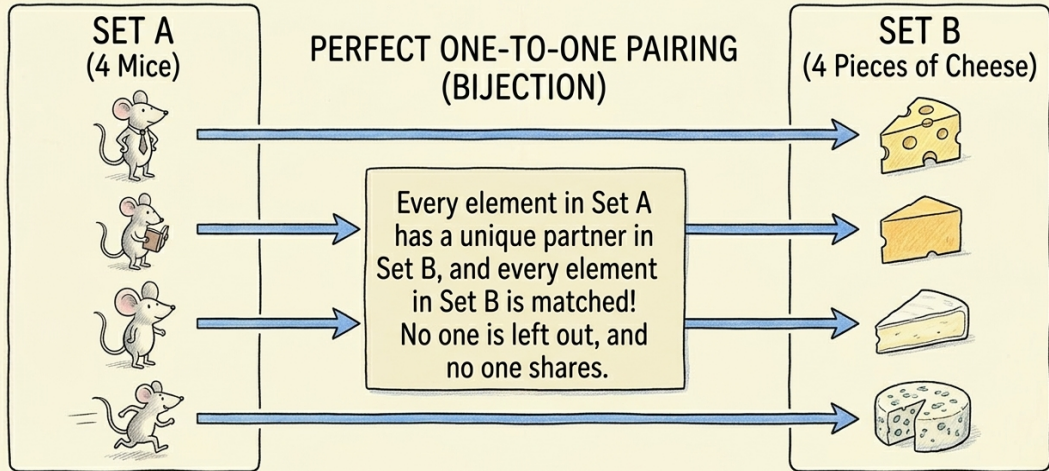
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- ▶ *Common understanding:* infinity = unreachable, or limitless, or very very every very large/far/big
- ▶ *Mathematical reality:* It is a deep subject connected to every inch of mathematical geography.
- ▶ *objective:* Move the public from a finite mindset which tries to imagine infinite to a transfinite one. To let people gain more power in imagination.

infinity

$0, 1, 2, \dots, 10^{10^{10^{10^{10}}}}, \dots, n, n+1, \dots, \aleph_0, \aleph_1, \aleph_{10^{10^{10}}}, \aleph_n, \dots, \aleph_{\aleph_0}, \aleph_{\aleph_n}, \aleph_{\aleph_{\aleph_n}}, \dots$

BIJECTION: VISUAL PROOF OF EQUINUMEROSITY FOR FINITE THINGS



CONCLUSION: Set A and Set B are EQUINUMEROUS (Same size: 4).

The perfect pairing IS the proof!

Gemini Pro 3.1



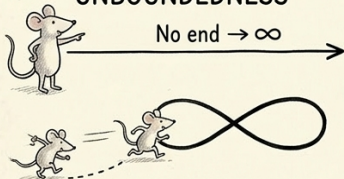


A SIMPLE GUIDE TO INFINITY (lines & mice!)

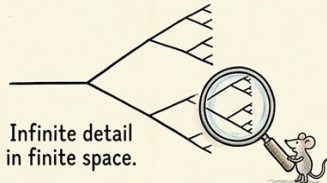


No complex equations allowed.

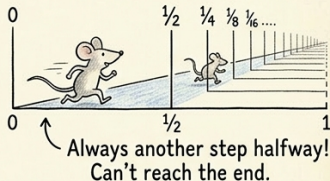
UNBOUNDEDNESS



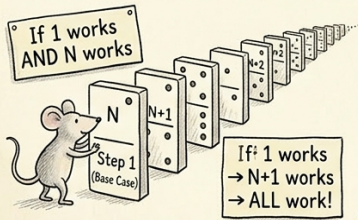
GEOMETRIC FRACTAL



DENSITY (The Halving Method)



MATHEMATICAL INDUCTION



SIZES OF INFINITY (The Big Challenge!)

(A) COUNTABLE

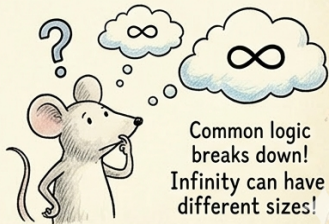


(B) UNCOUNTABLE



Mice numbers are countable,
lines are uncountably larger!
Gemini Pro 3.1

CHALLENGING INTUITION





INTEGERS = NATURAL NUMBERS

RATIONAL NUMBERS = NATURAL NUMBERS

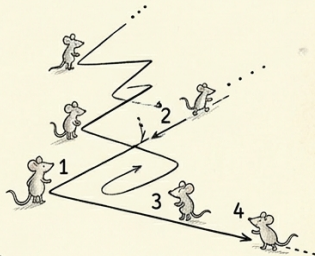


1  $\longrightarrow$ 0
2 $\longrightarrow$ 1
3  $\longrightarrow$ -1
4  $\longrightarrow$ 2
5...  $\longrightarrow$ -2...

Matching game: perfectly equinumerous!
Countable Infinity $\aleph_0$.


$$\begin{array}{ccccc} \frac{1}{1} & \frac{1}{2} & \frac{2}{1} & \frac{2}{2} & \frac{3}{1} & \dots \\ \frac{1}{1} & \frac{1}{3} & \frac{1}{3} & \frac{2}{2} & \frac{1}{1} & \dots \\ \frac{1}{1} & \frac{1}{2} & \frac{2}{3} & \frac{2}{4} & \frac{3}{1} & \dots \\ & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

We can list them all!
Also Countable $\aleph_0$.



REAL LINE > NATURAL NUMBERS

1 $\longrightarrow$ 0.a₁ a₂ a₃ ...
2 $\longrightarrow$ 0.b₁ b₂ b₃ ...
3 $\longrightarrow$ 0.b₁ b₂ b₃ ...
4 $\longrightarrow$ 0.a₁ a₂ a₃ ...
5... $\longrightarrow$ 0.e₁ e₂ e₃ ...

A magnifying glass with a wooden handle and a metal rim is positioned over the third row of the sequence, specifically focusing on the digits 3, 0, 2, and 6. The lens of the magnifying glass is centered over these four digits, which are slightly larger and more prominent than the others in the sequence. The handle of the magnifying glass extends towards the bottom right corner of the image.

Construct a new
"new" number by
changing
each
digit.



List always fails! Real Numbers are larger:
Uncountable Infinity $\mathfrak{C}$.



Gemini Pro 3.1

REAL LINE = REAL PLANE

$$\begin{aligned} x &= 0.123\dots \\ y &= 0.456\dots \end{aligned}$$
$$y = 0.456\dots$$

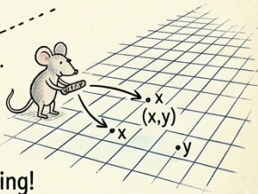
by

0.5

- 0.142536...

One-to-one mapping!

Same uncountably infinite number of points.



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activity: the infinite hotel manager



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Interactive theater (based on Hilbert's hotel).

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- ▶ The hotel is declared "*Full, yet we never turn a guest away.*"

activity: the infinite hotel manager

Interactive theater (based on Hilbert's hotel).

- ▶ A lobby area with a long hallway marked with room numbers $1, 2, 3 \dots$
- ▶ The hotel is declared "*Full, yet we never turn a guest away.*"
- ▶ *The logic in motion:*
 - ▶ $+1$ scenario: current guests move $n \rightarrow n + 1$.
 - ▶ Infinite bus ($\aleph_0$): current guests move $n \rightarrow 2n$.
 - ▶ ...

activity: the infinite hotel manager

Impact?

- ▶ Visitors physically experience *bijections* using their own bodies, while their mind tries to catch infinity.

$$P \stackrel{?}{=} NP$$

Core concepts: polynomial vs. exponential time, verification vs. search (P vs. NP).

- Are there problems that even the most powerful supercomputers can never solve?

$$P \stackrel{?}{=} NP$$

Core concepts: polynomial vs. exponential time, verification vs. search (P vs. NP).

- ▶ Are there problems that even the most powerful supercomputers can never solve?
- ▶ *Puzzle metaphor:*
 - ▶ Solving a 10,000-piece puzzle from scratch = *hard* (*search* / *NP*)
 - ▶ Recognizing the finished picture = *easy* (*verification* / *P*)

traveling salesperson

Hands-on competitive activity.

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- ▶ A physical map of 5 or 6 Polish cities with length-coded strings.

traveling salesperson

Hands-on competitive activity.

- ▶ A physical map of 5 or 6 Polish cities with length-coded strings.
- ▶ Challenge:
 - ▶ *the solver*: must find the absolute shortest route.
 - ▶ *the verifier*: simply checks if a proposed route is “under 500km.”

traveling sales-person

Impact?

traveling sales-person

Impact?

- ▶ Demystifying algorithms and explaining the mathematical barrier that makes digital cryptography work.

Smullyan's puzzle

Live logic puzzle (knights and knaves).

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- ▶ two doors guarded by a knight (truth) and a knave (lie).

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Solution:

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- ▶ two doors guarded by a knight (truth) and a knave (lie).
- ▶ find the correct door by asking exactly *one YES/NO question*.

Solution: *“if I asked the other guard if the left door is safe, would he say yes?”*

Smullyan's puzzle

Impact?

- ▶ Visitors practice truth-functional logic and deduction without writing a single equation. Moreover, they learn how to compose sentences which are truly logically complex and why some problems cannot have easy answers/solutions. (anti-populism)

fairness and cooperation

Core concepts: game theory, Nash equilibrium, Arrow's impossibility theorem.

- ▶ we assume that if everyone acts rationally, the best result will follow.

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- ▶ is “fairness” a mathematical guarantee?

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- ▶ we assume that if everyone acts rationally, the best result will follow.
- ▶ is “fairness” a mathematical guarantee?
- ▶ The objective is to demonstrate how logical systems can lead to paradoxes where the “rational” choice is the worst possible outcome.

voting and trust

Live experiments.

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- ▶ *the impossible election:*
 - ▶ tallying the exact same ballots using Plurality, Borda, and Condorcet.
 - ▶ *result:* different algorithms yield completely different winners.

voting and trust

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- ▶ *the impossible election:*
 - ▶ tallying the exact same ballots using Plurality, Borda, and Condorcet.
 - ▶ *result:* different algorithms yield completely different winners.
- ▶ *the paradox of trust (prisoner's dilemma):*
 - ▶ a high-stakes game of “share” vs “steal”.
 - ▶ *result:* individual rationality leads to a worse collective outcome.

the paradox of trust (prisoner's dilemma)

- ▶ a high-stakes game of “share” vs “steal”.
- ▶ *result*: individual rationality leads to a worse collective outcome.

Setup:

- ▶ two isolated suspects face a choice: stay silent (cooperate with each other) or confess (betray their partner).
- ▶ betraying your partner always yields a better individual reward, regardless of what the other person chooses to do.

paradox of trust (prisoner's dilemma)

It looks paradoxical, because perfectly logical self-interest drives both individuals to betray, they both end up with a worse outcome than if they had simply stayed silent.

paradox of trust (prisoner's dilemma)

It looks paradoxical, because perfectly logical self-interest drives both individuals to betray, they both end up with a worse outcome than if they had simply stayed silent. it proves that individual rationality does not always lead to the best collective outcomea foundational idea in economics, biology, and logic.

game table

The penalty table.

	B stays silent	B betrays
A stays silent	$A = B = 3Y$	$A = 13Y, B = 0$
A betrays	$A = 0, B = 13Y$	$A = B = 6Y$

Impact?

- ▶ Fairness is a complex logical puzzle, not just a feeling.

summary

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- ▶ We invite visitors to step beyond simply using everyday technology, empowering them to discover the clear reasoning that powers it.

summary

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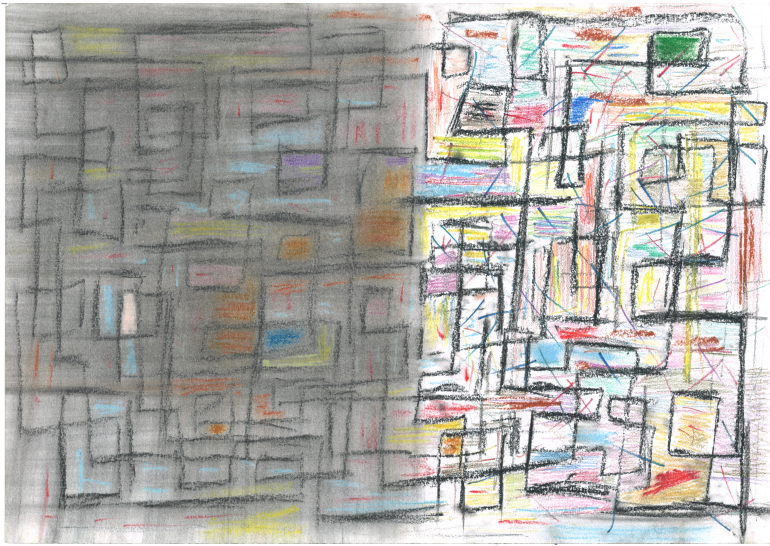
- ▶ We want to bring math to life. The logic lab turns the deep foundations of math into fun, hands-on experiences for everyone.
- ▶ We invite visitors to step beyond simply using everyday technology, empowering them to discover the clear reasoning that powers it.
- ▶ Our goal is to build a community that sees mathematics for what it truly is— a wonderful playground for the imagination.

gallery

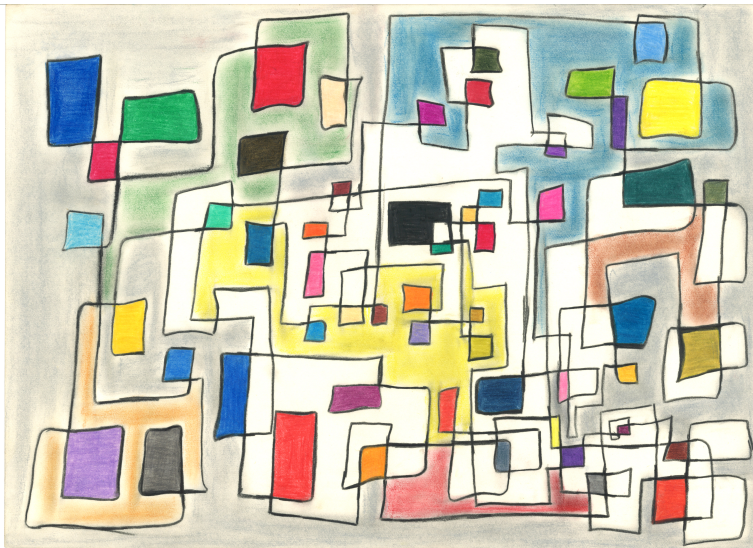
Some abstract paintings by myself



diagonalization, mixed media on paper, A3



complexity, mixed media on paper, A3



regularity, mixed media on paper, A3