The Story Of A Theorem

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TU Wien

ESTC 2022

Special session in honour of Boban Veličković

01.09.2022

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$$s <_T t \Rightarrow F(t) \neq F(s)$$

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- For $\mu \leq \kappa$, $\mathbb{S}_\mu(T)$ denotes the standard poset to generically specialise T with approximations of size less than μ.

- Lemma (BMR) If T is an Aronszajn tree, then $\mathbb{S}_\omega(T)$ has the countable chain condition.

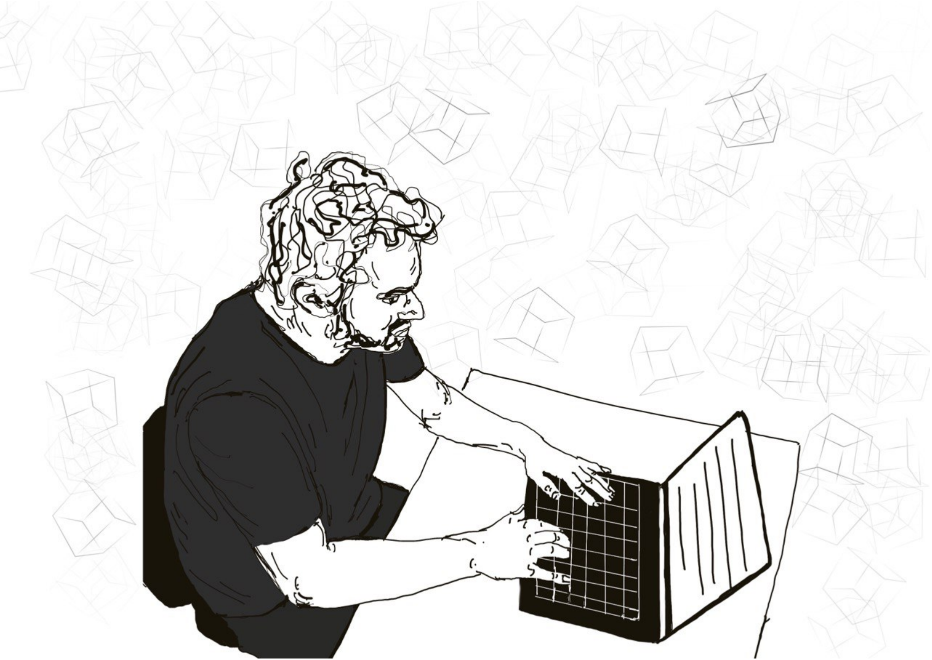
Those were the days


...Well, the proof is easy. We want to show that every condition is generic for ctble models. Things are finite, so you should be able to escape from incompatibility!
Okkk!

...Well, the proof is easy. We want to show that every condition is generic for stable models Things are finite, so you should be able to escape from incompatibility!
what f[redacted] kind of proof is this??!
Okkk!


...Well, the proof is easy. We want to show that every condition is generic for able models
Things are finite, so you should be able to escape from incompatibility!
What fabulous kind of proof is this??!
Okkk!

A helpless PhD student




Proceedings of the National Academy of Sciences
Vol. 67, No. 4, pp. 1748–1753, December 1970
Embedding Trees in the Rationals
J. Baumgartner,* J. Malitz, and W. Reinhardt
DARTMOUTH COLLEGE,* HANOVER, NEW HAMPSHIRE; AND DEPARTMENT OF MATHEMATICS, UNIVERSITY OF COLORADO, BOULDER, COLO. 80302
Communicated by S. Ulam, September 4, 1970
Abstract. An example is presented of a simple algebraic statement whose truth cannot be decided within the framework of ordinary mathematics, i.e., the statement is independent of the usual axiomatizations of set theory. The statement asserts that every tree-like ordering of power equal to or less than the first uncountable cardinal can be embedded homomorphically into the rationals.


This ridiculous proof doesn't resemble Boban's awful proof at all! Let's forget about both!
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A Revelation

The Two Cultures of Mathematics.

W. T. Gowers

In his famous Rede lecture of 1959, entitled "The Two Cultures", C. P. Snow argued that the lack of communication between the humanities and the sciences was very harmful, and he particularly criticized those working in the humanities for their lack of understanding of science. One of the most memorable passages draws attention to a lack of symmetry which still exists, in a milder form, forty years later:

Suppose

1. θ is an uncountable regular cardinal.
2. $M \prec H_\theta$ is countable.
3. Let $Z \in M$ a set. Suppose that $z \mapsto f_z$ is a function on $\mathcal{P}_{\omega_1}(Z)$ in M, where for each $z \in \mathcal{P}_{\omega_1}(Z)$, f_z is a $\{0,1\}$-valued function with $z \subseteq \mathrm{dom}(f_z)$.
4. $f : Z \cap M \to 2$ is a function that is not guessed in M.

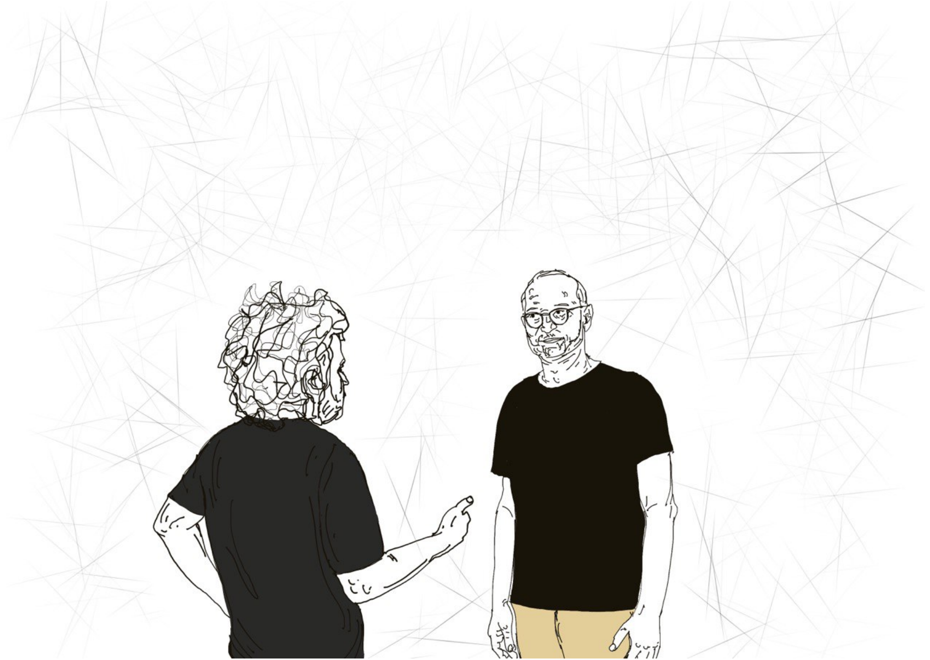
Assume that $B \in M$ is a cofinal subset of $\mathcal{P}_{\omega_1}(Z)$. Then there is $B^* \in M$ cofinal in B such that for every $z \in B^*$, $f_z \nsubseteq f$.

Suppose $\mathcal{I}$ is an ideal on a set Z. Let $M \prec H_\theta$ with $Z, \mathcal{I} \in M$. Let $f : M \cap W \to 2$ be a function which is not guessed in M, where $W \in M$. Suppose that $\phi \in M$ is a function such that:

1. $A = \mathrm{dom}(\phi) \in \mathcal{I}^+$,
2. For every $z \in A$, $\phi(z)$ is a function, and
3. For every $w \in W$, $\{z \in A : w \in \mathrm{dom}(\phi(z))\} \in \mathcal{I}^+$.

Then there is $A^* \in M \cap \mathcal{I}^+$ with $A^* \subseteq A$ such that for every $z \in A^*$, $\phi(z) \nsubseteq f$.

Le confinement parisien




I think I can specialise a branchless tree of height ω2 with finite conditions!


I think I can specialise a branchless tree of height ω_2 with finite conditions!
It's suspicious!


Why???


Why ???
Neeman has a paper in which he shows this is almost impossible you should check his paper to see why you don't contradict his result.

A jobless graduated student


Arch. Math. Logic (2017) 56:983–1036
DOI 10.1007/s00153-017-0550-y
Mathematical Logic
CrossMark
Two applications of finite side conditions at ω_2
Itay Neeman[1]
Received: 30 November 2013 / Accepted: 15 August 2015 / Published online: 27 May 2017
© Springer-Verlag Berlin Heidelberg 2017
Abstract We present two applications of forcing with finite sequences of models as side conditions, adding objects of size ω_2. The first involves adding a $\square_{\omega_1}$ sequence and variants of such sequences. The second involves adding partial weak specializing functions for trees of height ω_2.
Keywords Forcing · Finite conditions · Square · Specializing · Trees


hmmm! We have the same
forcing! Dang!
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Theorem(M.)

Assume PFA. Suppose that T is a branchless tree of height ω_2. Then there is a proper and ω_2-preserving forcing with the ω_1-approximation property $\mathbb{P}_T$ forcing T is special.

Now, let $p \in \mathbb{P}_T$ if and only if $p = (\mathcal{M}_p, f_p)$ satisfies:

1. $\mathcal{M}_p$ is a finite $\in$-chain of models in $\mathcal{C} \cup \mathcal{U}$, which is closed under intersections,
2. $f_p \in \mathbb{S}_\omega(T)$,

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4. for every countable $M \in \mathcal{M}_p$ and every $t \in \mathrm{dom}(f_p)$ with $f_p(t) \in M$, if t is "guessed" in M (i.e., t belongs to some branch $b \in M$), then $t \in M$.

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We say p is stronger than q if and only if:

1. $\mathcal{M}_p \supseteq \mathcal{M}_q$.
2. $f_p \supseteq f_q$.

Theorem(M.)

Let $\kappa = \kappa^{<\kappa}$. Assume that there are sufficiently many "nice" κ^+-sized κ^+-guessing models, and that there is an ideal on κ^{++} whose positive sets have a $<\kappa$-closed dense subset.

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Lemma

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Proof. Force with Neeman's or Velickovic's side conditions with decorations.


Happy Birthday!
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